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EFFECTS OF DIFFERENTIAL ROTATION ON THE MAXIMUM MASS OF NEUTRON STARS

Nicholas D. Lyford[1], Thomas W. Baumgarte [1,2] and Stuart L. Shapiro [2,3]
[1] Department of Physics and Astronomy, Bowdoin College, 8800 College Station Brunswick, ME 04011
[2] Department of Physics, University of Illinois at Urbana-Champaign, Urbana, IL 61801
[3] Department of Astronomy and NCSA, University of Illinois at Urbana-Champaign, Urbana, IL 61801
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ABSTRACT

The merger of binary neutron stars is likely to lead to differentially rotating remnants. In this paper we numerically construct models of differentially rotating neutron stars in general relativity and determine their maximum allowed mass. We model the stars adopting a polytropic equation of state and tabulate maximum allowed masses as a function of differential rotation and stiffness of the equation of state. We also provide a crude argument that yields a qualitative estimate of the effect of stiffness and differential rotation on the maximum allowed mass.

Subject headings: Gravitation — relativity — stars: rotation

1. INTRODUCTION

Neutron stars that are newly formed in the coalescence of binary neutron stars (or in supernova collapse) are likely to be differentially rotating (see, e.g., the fully dynamical simulations of Rasio & Shapiro 1992,1994, Shibata & Uryu 2000, Faber, Rasio & Manor 2001, as well as the review of Rasio & Shapiro 1999 and references therein). Differential rotation may play an important role for the stability of these remnants, since it can be very effective in increasing their maximum allowed mass.

Assuming that neutron stars in binaries have individual masses close to 1.4 $M_\odot$ (Thorsett & Chakrabarty 1999), and assuming that the maximum allowed mass of a nonrotating neutron star is in the range of $1.8 - 2.3\ M_\odot$ (Akmal, Pandharipande & Ravenhall 1998), one might conclude that the coalescence of binary neutron stars leads to immediate collapse to a black hole. However, both thermal pressure and rotation can increase the maximum allowed mass.

For coalescence from the innermost stable circular orbit, thermal pressure is believed to have a small effect. The maximum mass of *uniformly* rotating stars is limited by the spin rate at which the fluid at the equator moves on a geodesic; any further speed-up would lead to mass shedding (the "Kepler" limit). Uniform rotation can therefore increase the maximum allowed mass by at most about 20 % for very stiff equations of state (Cook, Shapiro & Teukolsky, 1992, hereafter CST1, and 1994, hereafter CST2), which is not sufficient to stabilize remnants of binary neutron star merger. Uniformly rotating equilibrium configurations with rest masses exceeding the maximum rest mass of nonrotating stars constructed with the same equation of state are referred to as "supramassive" stars (CST1, CST2).

Differential rotation, however, can be much more efficient in increasing the maximum allowed mass. In differentially rotating stars, the core may rotate faster than the envelope, so that the core can be supported by rapid rotation without the equator having to exceed the Kepler limit. This effect was demonstrated in Newtonian gravitation by Ostriker, Bodenheimer & Lynden-Bell (1966) for white dwarfs, and in general relativity by Baumgarte, Shapiro & Shibata (2000, hereafter BSS) for $n = 1$ polytropes. BSS also showed by way of example that stars with about 60 % more mass than the maximum allowed mass of the corresponding non-rotating star can be dynamically stable against both radial and nonaxisymmetric modes. BSS refer to differentially rotating equilibrium configurations with rest masses exceeding the maximum rest mass of a uniformly rotating star as "hypermassive" stars.

In this paper we extend the findings of BSS for $n = 1$ polytropes in two ways. We survey different polytropic indices, and study the effect of both differential rotation and stiffness of the equation of state on the maximum allowed mass. We also introduce a very simple model calculation that illustrates these effects qualitatively, and that moreover gives surprisingly accurate results for soft equations of state and moderate degrees of differential rotation.

This paper is organized as follows. In Section 2 we present qualitative considerations, leading to a simple estimate of the maximum mass of differentially rotating neutron stars. In Section 3 we construct numerical models of fully relativistic, differentially rotating neutron stars for different polytropic indices. We briefly discuss our findings in Section 4. We also include Appendix A with tables of our numerical results.

2. QUALITATIVE CONSIDERATIONS

When magnetic fields and relativistic effects can be neglected, rotating stars in equilibrium satisfy the Newtonian virial theorem

$$W + 2T + 3\Pi = W\left(1 - 2\frac{T}{|W|}\right) + 3\Pi = 0 \qquad (1)$$

(see, e.g., Shapiro & Teukolsky 1983). Here the rotational kinetic energy T scales as

$$T \sim \frac{J^2}{MR_e^2}, \qquad (2)$$

the potential energy as

$$W \sim -\frac{M^2}{R_e} \sim -M^{5/3}\rho^{1/3}, \qquad (3)$$

and the internal energy, computed from a volume integral of the pressure, as

$$\Pi \sim \rho^{1/n} M. \tag{4}$$

where J is the angular momentum, M the mass, R_e the equatorial radius and ρ the mass density. We have also assumed a polytropic equation of state

$$P = K\rho^{1+1/n}, \tag{5}$$

where K is a constant and n the polytropic index. Here and throughout we set the gravitation constant $G = 1$.

Inserting the above expressions into the virial equation (1), we find

$$-\beta M^{5/3}\rho^{1/3}\left(1 - 2\frac{T}{|W|}\right) + \gamma\rho^{1/n} M = 0, \tag{6}$$

where β and γ are the appropriate coefficients. In general, β depends on the eccentricity of the star, but restricting our analysis to small values of $T/|W|$, and hence to nearly spherical stars, we assume that both β and γ are constant.

For non-rotating stars ($T = 0$) we solve equation (6) to find

$$M = \left(\frac{\gamma}{\beta}\right)^{3/2} \rho^{(3-n)/(2n)}, \tag{7}$$

while for rotating stars we obtain

$$M_{\rm rot} = M\left(1 - 2\frac{T}{|W|}\right)^{-3/2}. \tag{8}$$

For small values of $T/|W|$ the right hand side can be expanded, and we then find for the mass increase $\delta M \equiv M_{\rm rot} - M$

$$\frac{\delta M}{M} = 3\frac{T}{|W|} \tag{9}$$

(cf. Shapiro & Teukolsky 1983, eqn 7.4.40). This expression determines the fractional mass increase as a function of $T/|W|$ for a constant value of the density ρ. We will use this result to estimate the increase in the maximum allowed mass of a neutron star, even though typically rotating stars assume their maximum masses at slightly different densities than the corresponding non-rotating stars.

To evaluate T and $|W|$ we further simplify the problem by assuming that the star's density profile is a step function with a constant density ρ_c (equal to the original central density) inside a spherical core of radius R_c, and zero outside. From requiring that the mass of this model star,

$$M = \frac{4\pi}{3}\rho_c R_c^3, \tag{10}$$

be equal to the original mass

$$M = \frac{4\pi}{3}\bar{\rho} R_e^3, \tag{11}$$

where $\bar{\rho}$ is the average density, we find the following relation between the central condensation and the ratio of the radii

$$\frac{R_e}{R_c} = \left(\frac{\rho_c}{\bar{\rho}}\right)^{1/3}. \tag{12}$$

Further assuming that the core is uniformly rotating with the central angular velocity Ω_c, we find

$$T = \frac{1}{2} I \Omega_c^2 = \frac{M R_c^2}{5}\Omega_c^2. \tag{13}$$

Inserting this relation together with the potential energy

$$W = -\frac{3}{5}\frac{M^2}{R_c}. \tag{14}$$

into (9) yields

$$\frac{\delta M}{M} \sim \frac{R_c^3 \Omega_c^2}{M}. \tag{15}$$

To find the increase in the maximum allowed mass, it is useful to assume that the star rotates at the mass-shedding limit

$$\Omega_e^2 = \frac{M}{R_e^3}, \tag{16}$$

where Ω_e is the equatorial angular velocity. This equation can now be used to eliminate the mass M in equation (15), which, together with (12), yields

$$\frac{\delta M}{M} \sim \frac{\bar{\rho}}{\rho_c}\left(\frac{\Omega_c}{\Omega_e}\right)^2. \tag{17}$$

Equation (17) provides a very simple estimate for the increase of the maximum allowed mass. It depends only on the central condensation of the non-rotating star, which is a function of the stiffness of the equation of state, and the ratio of the angular velocities at the center and equator, which is a function of the the degree of differential rotation. For uniformly rotating stars, the maximum mass increase is estimated to be simply the inverse of the central condensation. In Table 1 we compare this estimate with the numerical findings of CST2 and find remarkably good agreement for soft equations of state. Table 1 also illustrates an ambiguity; in Newtonian gravity, the central condensation is uniquely determined by the polytropic index, but in general relativity the central condensation of a star depends on the central density. We therefore compute a "relativistic" central condensation $\rho_c/\bar{\rho}|_{\rm TOV}$ from the central energy density ρ_c and an average density defined as

$$\bar{\rho} = \frac{3M}{4\pi R^3} \tag{18}$$

of the non-rotating maximum mass model, where M is the total mass-energy of the star, and R is the circumferential radius. We find that this value yields better agreement with the numerical values of the maximum mass increase than adopting the Newtonian central condensation.

The ratio $T/|W|$ provides a useful criterion for the onset of secular ($T/|W| \sim 0.14$) or dynamical ($T/|W| \sim 0.27$) non-axisymmetric instabilities. Inserting these limits into equation (8) shows that mass increases of secularly stable stars are limited by $\delta M/M \lesssim 1.63$ while mass increases of dynamically stable stars are limited by $\delta M/M \lesssim 3.2$. These values agree quite well with the respective limits of 1.70 and 3.51 found by Shapiro & Teukolsky (1983; equations (7.4.41) and (7.4.42)), who also take stellar deformations into account.

TABLE 1
MAXIMUM MASS INCREASE FOR UNIFORMLY ROTATING POLYTROPES.

n [a]	$\bar{M}_0^{\max}$ [b]	$\bar{R}^{\max}$ [c]	$\bar{\rho}_{\max}$ [d]	$\delta M/M\|_{\rm CST}$ [e]	$\bar{\rho}/\rho_c\|_{\rm TOV}$ [f]	$\bar{\rho}/\rho_c\|_{\rm Newt}$ [g]
0.5	0.151	0.395	1.29	0.224	0.375	0.545
1.0	0.180	0.763	0.42	0.146	0.209	0.304
1.5	0.276	1.97	0.072	0.099	0.115	0.167
2.0	0.523	6.94	5.8×10^{-3}	0.066	0.063	0.088
2.5	1.25	41.0	1.26×10^{-4}	0.040	0.034	0.043
2.9	3.23	620	1.54×10^{-7}	0.023	0.021	0.021

[a] Polytropic index.
[b] Maximum rest mass of non-rotating polytrope (CST2).
[c] Circumferential radius of the non-rotating maximum-mass configuration (CST2).
[d] Maximum energy density of the non-rotating maximum mass configuration (CST2).
[e] Fractional rest mass increase (CST2).
[f] Estimate (17) using relativistic central condensation.
[g] Estimate (17) using Newtonian central condensation.

3. NUMERICAL RESULTS

We use a modified version of the numerical code of CST1 and CST2 to construct models of differentially rotating neutron stars. The code is based on similar algorithms developed by Hachisu (1986) and Komatsu, Eriguchi and Hachisu (1989), and we refer to CST1 for details. We adopt a polytropic equation of state

$$P = K\rho_0^{1+1/n}, \tag{19}$$

where ρ_0 is the rest-mass density and where equation (19) reduces to equation (5) in the Newtonian limit. We take the polytropic constant K to be unity without loss of generality. Since $K^{n/2}$ has units of length, all solutions scale according to $\bar{M} = K^{-n/2}M$, $\bar{\rho}_0 = K^n\rho_0$, etc., where the barred quantities are dimensionless quantities corresponding to $K = 1$, and the unbarred quantities are physical quantities (compare CST1).

Constructing differentially rotating neutron star models requires choosing a rotating law $F(\Omega) = u^t u_\phi$, where u^t and u_ϕ are components of the four-velocity u^α and Ω is the angular velocity. We follow CST1 and assume the rotation law $F(\Omega) = A^2(\Omega_c - \Omega)$, where the parameter A has units of length. Expressing u^t and u_ϕ in terms Ω and metric potentials yields equation (42) in CST1, or, in the Newtonian limit,

$$\Omega = \frac{\Omega_c}{1 + \hat{A}^{-2}\hat{r}^2\sin^2\theta}. \tag{20}$$

Here we have rescaled A and r in terms of the equatorial radius R_e: $\hat{A} \equiv A/R_e$ and $\hat{r} \equiv r/R_e$. The parameter $\hat{A}$ is a measure of the degree of differential rotation and determines the length scale over which Ω changes. Since uniform rotation is recovered in the limit $\hat{A} \to \infty$, it is convenient to parametrize sequences by $\hat{A}^{-1}$. In the Newtonian limit, the ratio Ω_c/Ω_e that appears in the estimate (17) is related to $\hat{A}^{-1}$ by $\Omega_c/\Omega_e = 1 + \hat{A}^{-2}$, but for relativistic configurations this relation holds only approximately.

We adopt this particular rotation law for convenience and for easy comparison with many other authors who have assumed the same law. We also compared with the remnants' angular momentum distribution in the fully relativistic dynamical merger simulations of Shibata and Uryu (2000) and to the post-Newtonian simulations of Faber, Rasio and Manor (2001). We have found that their numerical results can be fit reasonably well by our adopted differential rotation law.

We modify the numerical algorithm of CST1 by fixing the maximum interior density instead of the central density for each model. This change allows us to construct higher mass models in some cases, since the central density does not always coincide with the maximum density and hence may not specify a model uniquely.

For a given a value of n and $\hat{A}$, we construct a sequence of models for each value of the maximum density by starting with a static, spherically symmetric star and then decreasing the ratio of the polar to equatorial radius, $R_{pe} = R_p/R_e$, in decrements of 0.025. This sequence ends when we reach mass shedding (for large values of $\hat{A}$), or when the code fails to converge (indicating the termination of equilibrium solutions) or when $R_{pe} = 0$ (beyond which the star would become a toroid). For each one of these sequences the maximum achieved mass is recorded. We repeat this procedure for different values of the maximum density, covering about a decade below the central density of the non-rotating maximum mass model, which yields the maximum allowed mass for the chosen values of n and $\hat{A}$. Our numerical results are tabulated in Appendix A. Our maximum mass increases are lower limits in the sense that even higher mass models may exist, but that we have not been able to construct them numerically.

4. DISCUSSION AND SUMMARY

Our numerical results are tabulated in Tables A2 to A9 in Appendix A. We also compare the increases in the maximum allowed mass with the estimate (17), and find surprisingly good agreement for soft equations of state and moderate degrees of differential rotation.

In particular, we find that the fractional maximum rest mass increase $\delta M/M$ for uniformly rotating stars is well approximated by the inverse of the central concentration (see also Table 1). For moderate degrees of differential

rotation, $\delta M/M$ increases approximately with the square of the ratio between the central and equatorial angular velocity Ω_c/Ω_e, in accord with equation (17).

For all equations of state we find that $\delta M/M$ increases with Ω_c/Ω_e only up to a moderate value of Ω_c/Ω_e, and starts to decrease again for larger values (at least with our code and algorithm we do not find monotonically increasing mass configurations). For stiff equations of state this turn-around occurs for smaller values of Ω_c/Ω_e than for soft equations of state. For moderate degrees of differential rotation $\Omega_c/\Omega_e \lesssim 2$, a given value of Ω_c/Ω_e will lead to a larger increase in the maximum allowed mass for a stiffer equation of state, as expected from the estimate (17).

We find the largest maximum mass increases for moderately stiff equations of state. Some of these configurations exceed the maximum allowed mass of the corresponding non-rotating star by more than a factor of two. These configurations typically have large values of $T/|W| \gtrsim 0.27$, indicating that such stars may by dynamically unstable against bar formation (but see Shibata, Karino & Eriguchi 2002, who found mild bar mode instabilities at very small values of $T/|W|$ for extreme degress of differential rotation). They are also "toriodal", i.e. assume their maximum density on a torus around the center of the star, which may indicate an $m = 1$ instability at even smaller values of $T/|W|$ (Centrella, New, Lowe & Brown 2001). However, even restricting attention to those configurations that are not toriodal and have $T/|W| < 0.27$, we find configurations with masses larger than the maximum mass of the corresponding non-rotating star by over 60 %. BSS demonstrated that at least some of these models are dynamically stable. Shibata & Uryu (2000) demonstrated that binary mergers may result in similarly stable hypermassive stars, when the progenitor masses are not too close to the maximum mass.

To summarize, we find that differential rotation is very effective in increasing the maximum allowed mass, especially for moderately stiff equations of state. The effect is probably large enough to stabilize the remnants of binary neutron star merger, which are likely to be differentially rotating. Binary neutron star coalescence may therefore lead to secularly stable, hypermassive neutron stars. As discussed in BSS (see also Shapiro 2000), magnetic braking is likely to bring such differentially rotating stars into uniform rotation, which reduces the maximum allowed mass and induces a delayed collapse to a Kerr black hole.

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APPENDIX

NUMERICAL RESULTS FOR MAXIMUM MASSES

We list below in Tables A2 to A9 values for the maximum rest mass increase for uniformly and differentially rotating polytropes. For each polytropic index n we tabulate the differential rotation parameter $\hat{A}^{-1}$, the ratio of the central and equatorial angular velocity Ω_c/Ω_e (which reduces to (20) in the Newtonian limit, i.e. for soft equations of state), the numerically determined fractional rest mass increase $(\delta M/M)_{\rm num}$, the ratio of the (relativistic) rotational kinetic energy and the gravitational binding energy $T/|W|$, the ratio between polar and equatorial radius R_p/R_e, the maximum density $\bar{\rho}_{\rm max}$, and the estimate $(\delta M/M)_{\rm est}$ according to equation (17). In these estimates we used the numerically determined ratios Ω_c/Ω_e and the central condensations according to (18). For $n \leq 1.25$ some of the maximum mass configurations are toriodal, i.e. assume the maximum density on a toroid about the center. For these polytropic indices we also include the ratio $\rho_c/\rho_{\rm max}$.

All models are computed with the code of CST1 and CST2, using 64 zones both in the radial and angular direction, and truncating the Legendre polynomial expansion at $\ell = 16$ (see CST1 for details of the numerical implementation). The accuracy of individual stellar models can be tested, for example, by computing a relativistic Virial theorem (Gourgoulhon & Bonazzola 1994; Cook, Shapiro & Teukolsky 1996; see also Nozawa et al. 1998 for a comparison of several different computational methods). In our analysis, however, the error in the maximum mass and related quantities is dominated by the finite step size in the sequences over R_p/R_e and $\bar{\rho}_{\rm max}$, which result in errors typically in the order of a few percent. For soft equations of state the mass as a function of central density is a very slowly varying function, making it quite difficult to determine the central density of the maximum mass configuration very accurately. The error in the R_p/R_e is determined by our stepsize of 0.025. We finally note that highly toroidal configurations depend very sensitively on the input parameters, so that those mass increases should only be taken as estimates.

Table A2

$n = 0.5$: $M_0^{\rm max} = 0.151$; $\bar{\rho}/\rho_c|_{\rm TOV} = 0.375$.

$\hat{A}^{-1}$	Ω_c/Ω_e	$(\delta M/M)_{\rm num}$	$T/\|W\|$	R_p/R_e	$\bar{\rho}_{\rm max}$	$\rho_{\rm max}/\rho_c$	$(\delta M/M)_{\rm est}$
0.0	1.00	0.22	0.15	0.55	1.02	1	0.38
0.3	1.51	0.41	0.23	0.425	0.73	1	0.86
0.5	1.93	0.62	0.31	0.2	0.30	0.77	1.40
0.7	2.46	0.46	0.30	0.025	0.29	0.085	2.27
1.0	3.54	0.20	0.27	0.1	0.31	0.29	4.70

TABLE A3

$n = 0.75$: $M_0^{\max} = 0.159$; $\bar{\rho}/\rho_c|_{\rm TOV} = 0.28$.

$\hat{A}^{-1}$	Ω_c/Ω_e	$(\delta M/M)_{\rm num}$	$T/\lvert W\rvert$	R_p/R_e	$\bar{\rho}_{\max}$	$\rho_{\max}/\rho_c$	$(\delta M/M)_{\rm est}$
0.0	1.00	0.18	0.11	0.575	0.67	1	0.28
0.3	1.35	0.27	0.15	0.5	0.64	1	0.52
0.5	1.92	0.51	0.22	0.4	0.42	1	1.04
0.7	2.49	1.07	0.30	0.025	0.16	0.021	1.75
1.0	3.48	0.68	0.27	0.025	0.16	0.019	3.42
1.5	6.33	0.19	0.16	0.425	0.27	0.74	11.3

TABLE A4

$n = 1$: $M_0^{\max} = 0.180$; $\bar{\rho}/\rho_c|_{\rm TOV} = 0.209$.

$\hat{A}^{-1}$	Ω_c/Ω_e	$(\delta M/M)_{\rm num}$	$T/\lvert W\rvert$	R_p/R_e	$\bar{\rho}_{\max}$	$\rho_{\max}/\rho_c$	$(\delta M/M)_{\rm est}$
0.0	1.00	0.15	0.083	0.575	0.35	1	0.21
0.3	1.24	0.20	0.10	0.55	0.33	1	0.32
0.5	1.65	0.31	0.14	0.475	0.32	1	0.57
0.7	2.33	0.61	0.21	0.375	0.23	1	1.14
0.8	2.66	1.12	0.28	0.25	0.083	0.65	1.48
0.85	2.78	1.40	0.29	0.025	0.068	5.0×10^{-3}	1.60
1.0	3.39	1.22	0.28	0.025	0.075	4.7×10^{-3}	2.28
1.5	6.33	0.31	0.15	0.475	0.23	0.81	8.37

TABLE A5

$n = 1.25$: $M_0^{\max} = 0.216$; $\bar{\rho}/\rho_c|_{\rm TOV} = 0.15$.

$\hat{A}^{-1}$	Ω_c/Ω_e	$(\delta M/M)_{\rm num}$	$T/\lvert W\rvert$	R_p/R_e	$\bar{\rho}_{\max}$	$\rho_{\max}/\rho_c$	$(\delta M/M)_{\rm est}$
0.0	1.00	0.12	0.063	0.6	0.15	1	0.15
0.3	1.19	0.16	0.075	0.575	0.15	1	0.21
0.5	1.51	0.21	0.10	0.525	0.15	1	0.33
0.7	1.98	0.32	0.13	0.475	0.14	1	0.58
1.0	3.39	1.78	0.28	0.025	0.037	1.1×10^{-3}	1.69
1.5	5.26	0.27	0.12	0.575	0.101	0.99	4.10

TABLE A6

$n = 1.5$: $M_0^{\max} = 0.276$; $\bar{\rho}/\rho_c|_{\rm TOV} = 0.115$.

$\hat{A}^{-1}$	Ω_c/Ω_e	$(\delta M/M)_{\rm num}$	$T/\lvert W\rvert$	R_p/R_e	$\bar{\rho}_{\max}$	$(\delta M/M)_{\rm est}$
0.0	1.00	0.10	0.047	0.625	0.061	0.12
0.3	1.15	0.12	0.055	0.6	0.060	0.15
0.5	1.42	0.16	0.068	0.575	0.059	0.23
0.7	1.81	0.22	0.089	0.525	0.056	0.38
1.0	2.65	0.40	0.137	0.45	0.047	0.81
1.5	4.75	0.25	0.098	0.625	0.050	2.60

TABLE A7

$n = 2.0$: $M_0^{\max} = 0.523$; $\bar{\rho}/\rho_c|_{\rm TOV} = 0.063$.

$\hat{A}^{-1}$	Ω_c/Ω_e	$(\delta M/M)_{\rm num}$	$T/\lvert W\rvert$	R_p/R_e	$\bar{\rho}_{\max}\times10^3$	$(\delta M/M)_{\rm est}$
0.0	1.00	0.067	0.027	0.65	5.1	0.063
0.3	1.12	0.076	0.031	0.625	5.1	0.079
0.5	1.33	0.092	0.036	0.625	5.1	0.11
0.7	1.63	0.12	0.045	0.6	4.9	0.17
1.0	2.28	0.18	0.064	0.55	4.4	0.33
1.5	4.03	0.15	0.059	0.7	7.0	1.03

TABLE A8
$n = 2.5$: $M_0^{\max} = 1.25$; $\bar{\rho}/\rho_c|_{\rm TOV} = 0.034$.

$\hat{A}^{-1}$	Ω_c/Ω_e	$(\delta M/M)_{\rm num}$	$T/\|W\|$	R_p/R_e	$\bar{\rho}_{\max} \times 10^4$	$(\delta M/M)_{\rm est}$
0.0	1.00	0.043	0.016	0.675	1.15	0.034
0.3	1.10	0.048	0.017	0.65	1.15	0.041
0.5	1.28	0.056	0.020	0.65	1.1	0.056
0.7	1.54	0.069	0.024	0.625	1.1	0.081
1.0	2.10	0.098	0.034	0.6	1.1	0.15
1.5	3.50	0.102	0.035	0.75	1.05	0.42
2.0	5.44	0.053	0.019	0.875	1.1	1.01

TABLE A9
$n = 2.9$: $M_0^{\max} = 3.23$; $\bar{\rho}/\rho_c|_{\rm TOV} = 0.021$.

$\hat{A}^{-1}$	Ω_c/Ω_e	$(\delta M/M)_{\rm num}$	$T/\|W\|$	R_p/R_e	$\bar{\rho}_{\max} \times 10^7$	$(\delta M/M)_{\rm est}$
0.0	1.00	0.028	0.010	0.675	1.4	0.021
0.3	1.09	0.031	0.011	0.675	1.3	0.025
0.5	1.25	0.037	0.013	0.675	1.3	0.033
0.7	1.50	0.044	0.015	0.65	1.3	0.047
1.0	2.02	0.062	0.020	0.625	1.3	0.085
1.5	3.29	0.069	0.022	0.775	1.3	0.227
2.0	5.07	0.035	0.012	0.9	1.4	0.540

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